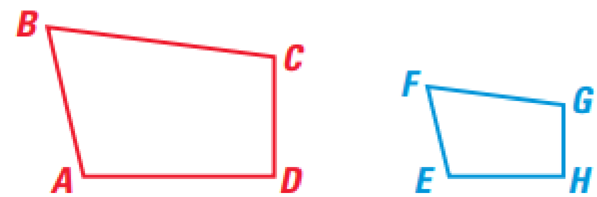
7.3 Similar Right Triangles

Vocabulary!!

- **Similar polygons –** polygons whose corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram to the right, *ABCD* is similar to *EFGH* and can be written as

$ABCD \sim EFGH$



Corresponding angles:

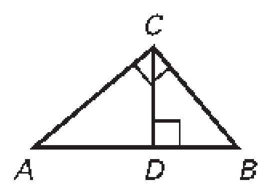
$\angle A \cong \angle E, \angle B \cong \angle F, \angle C \cong \angle G, \angle D \cong \angle H$

Ratios of corresponding sides:

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH}$$

THEOREM 9.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

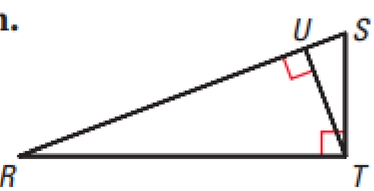


$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

similarity statement

Example 1: *Identifying Similar Triangles*

Identify the similar triangles in the diagram.



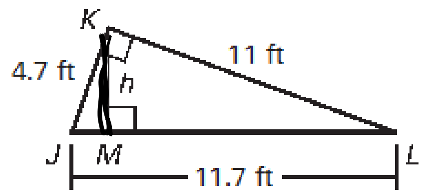
Solution

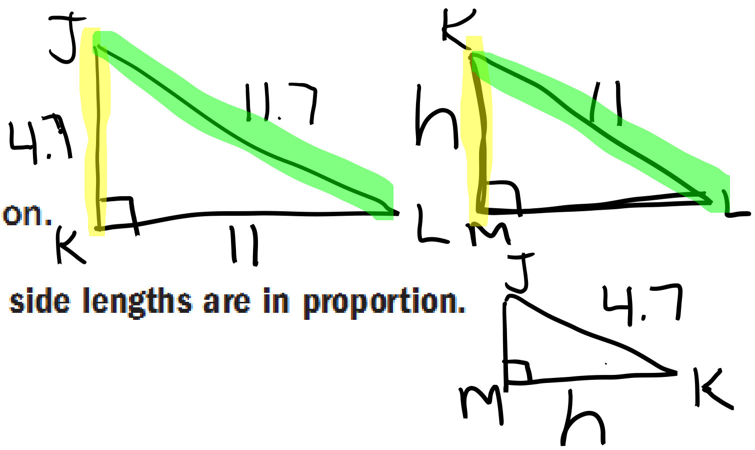
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

$\triangle RTS \sim \triangle RUT \sim \triangle TUS$

Example 2: *Finding the Height of a Ramp*

Ramp Height A ramp has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the ramp.





Solution

By Theorem 9.1, $\triangle JKL \sim \triangle KML$.

Use similar triangles to write a proportion.

$$\frac{KM}{\boxed{JK}} = \frac{KL}{JL}$$ **Corresponding side lengths are in proportion.**

$$\frac{h}{\boxed{4.7}} = \frac{11}{11.7}$$ **Substitute.**

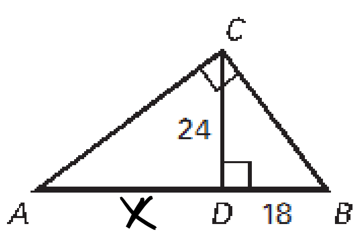
$$11.7h = 11(4.7)$$ **Cross product property**

$$h \approx 4.4$$ **Solve for *h*.**

Answer The height of the ramp is about 4.4 feet.

Checkpoint **Write similarity statements for the three triangles in the diagram. Then find the given length. Round decimals to the nearest tenth, if necessary.**

1. Find AD.



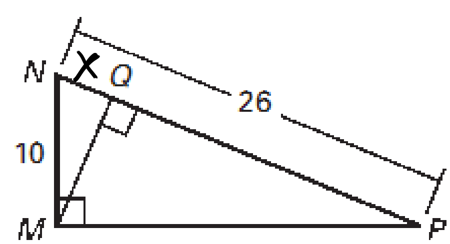
$\triangle \cancel{ABC} \sim \triangle ACD \sim \triangle CBD$

$$\frac{AD}{CD} = \frac{CD}{BD} \rightarrow \frac{x}{24} = \frac{24}{18}$$

$576 = 18x$

$x = 32$

2. Find NQ.



$\triangle NMP \sim \triangle \cancel{MQP} \sim \triangle NQM$

$$\frac{NQ}{NM} = \frac{NM}{NP} \rightarrow \frac{x}{10} = \frac{10}{26}$$

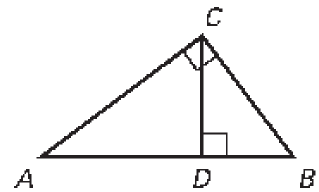
$100 = 26x$

$x = 3.8$

GEOMETRIC MEAN THEOREMS

THEOREM 9.2

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.



The length of the altitude is the geometric mean of the lengths of the two segments.

$$\frac{BD}{\boxed{CD}} = \frac{\boxed{CD}}{AD}$$

THEOREM 9.3

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

$$\frac{AB}{CB} = \frac{CB}{\boxed{DB}}$$

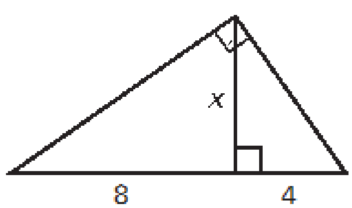
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$$\frac{AB}{AC} = \frac{AC}{\boxed{AD}}$$

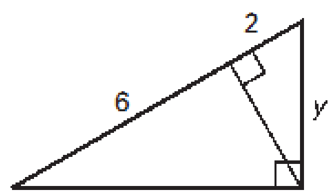
Example 3: ***Using a Geometric Mean***

Find the value of each variable.

a.



b.



Solution

a. Apply Theorem 9.2.

$$\frac{\boxed{4}}{x} = \frac{x}{8}$$

$$32 = x^2$$

$$\sqrt{32} = x$$

$$4\sqrt{2} = x$$

b. Apply Theorem 9.3.

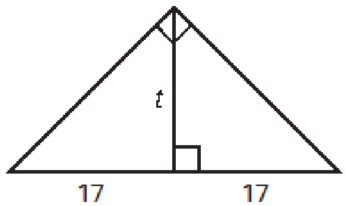
$$\frac{8}{y} = \frac{y}{\boxed{2}}$$

$$16 = y^2$$

$$4 = y$$

✔ *Checkpoint* **Find the value of the variable.**

3.

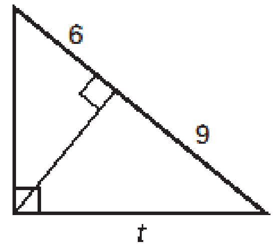


$$\frac{t}{17} = \frac{17}{t}$$

$$t^2 = 17^2$$

$$t = 17$$

4.



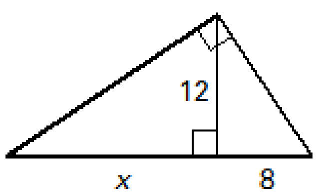
$$\frac{15}{t} = \frac{t}{9}$$

$$135 = t^2$$

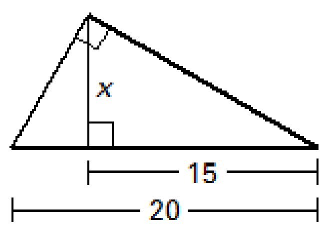
$$t = 3\sqrt{15}$$

Complete and solve the proportion.

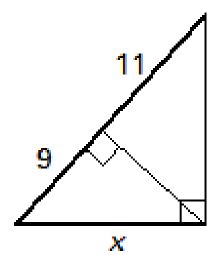
1. $\frac{x}{12}$ $\frac{?}{8}$



2. $\frac{15}{x}$ $\frac{x}{?}$

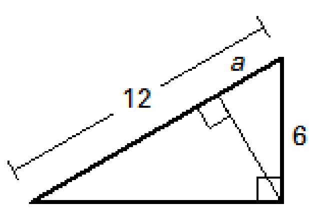


3. $\frac{9}{x}$ $\frac{x}{?}$

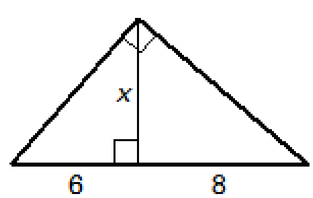


Find the value(s) of the variable(s).

4.



5.



6.